

Wallisdean Junior School

Calculation Policy



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Aims and Rationale

Children develop their processes of calculation through the CPA approach (concrete, pictorial and abstract). As children's methods are strengthened and refined, so too are their informal written methods. These methods develop to more abstract and efficient written methods. The use of CPA methods are important in ensuring that children fully understand the process of calculations and what happens to the numbers and why. At the beginning, the children will begin by using concrete and oral methods. They will move towards using pictorial methods alongside these to consolidate their understanding, eventually moving towards abstract written methods. It is important that children go through this process of methods when learning any new topic in mathematics to ensure that they fully understand the process of the calculation.

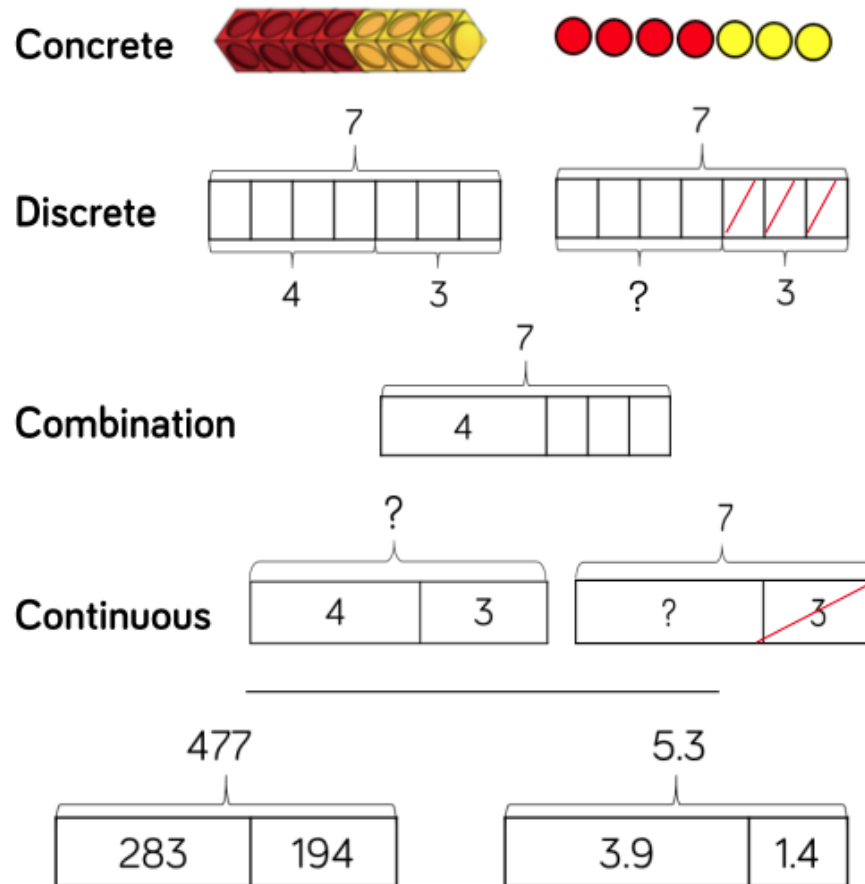
The calculation policy will:

- ensure consistency and progression across the school in the teaching and learning of calculation skills using the four operations.
- promote conceptual understanding, building on models and images using the CPA approach.
- provide guidance on the teaching of calculation skills.

Using the calculation policy

The calculation policy is organised into two main sections: addition and subtraction and multiplication and division. At the start of each section, there is an overview of the different models and images that can support the teaching of the different concepts. Each operation is then broken down into skills and different models and images that could be used to effectively teach that concept. At the end of the calculation policy there is a glossary to explain key mathematical terms.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

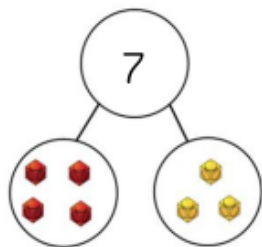
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

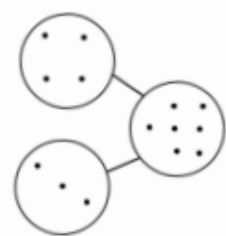
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Part-Whole Model



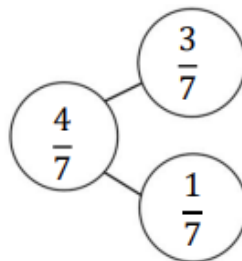
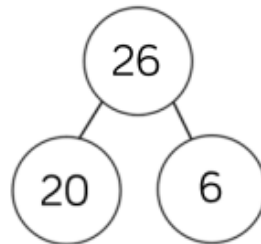
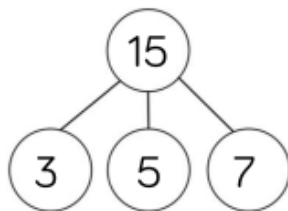
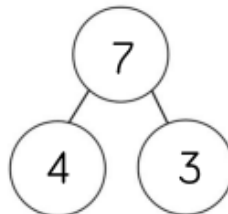
$$7 = 4 + 3$$

$$7 = 3 + 4$$



$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

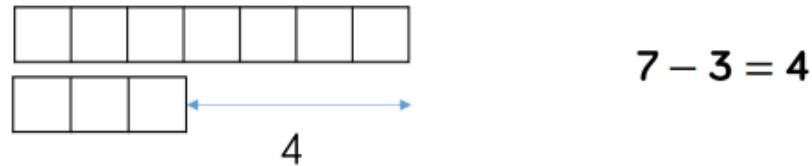
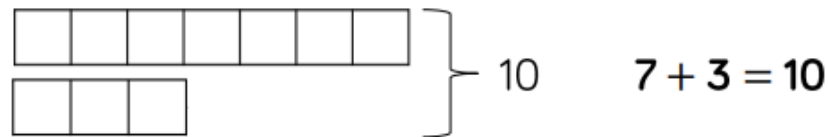
When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (multiple)

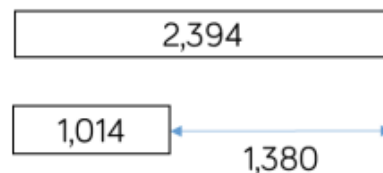
Discrete



Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

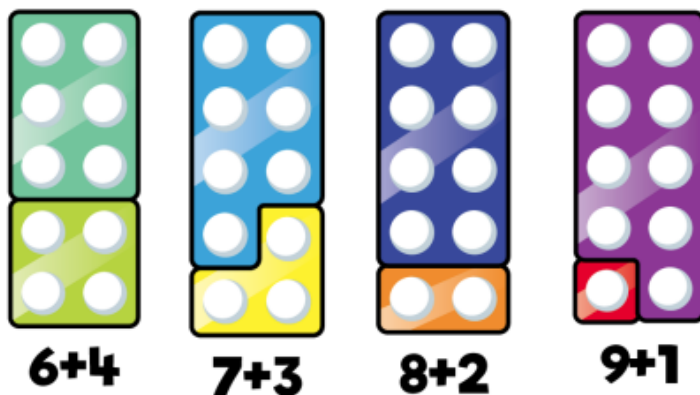
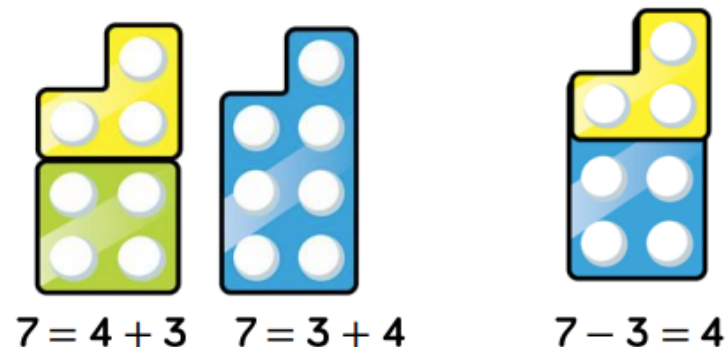
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

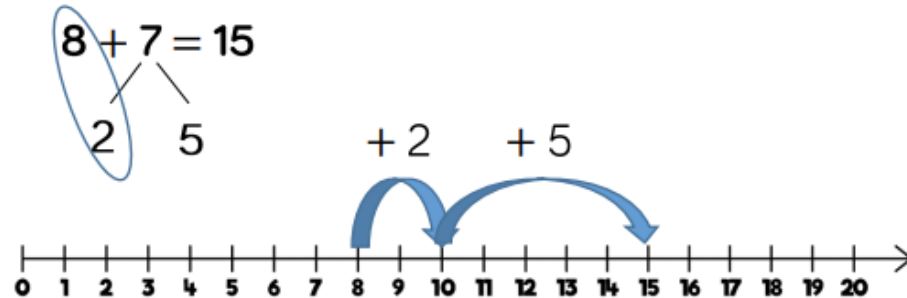
Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Number Lines (labelled)

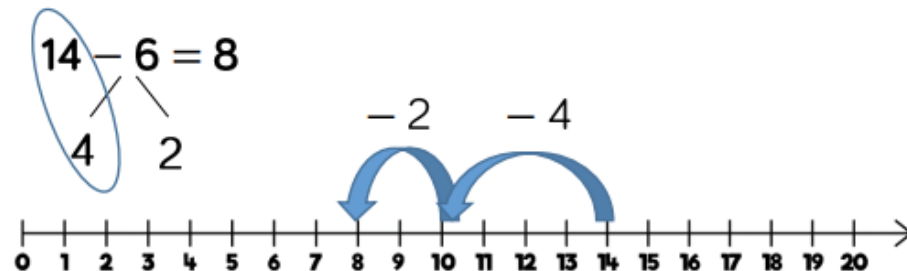
$$5 + 3 = 8$$



$$8 + 7 = 15$$



$$14 - 6 = 8$$



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

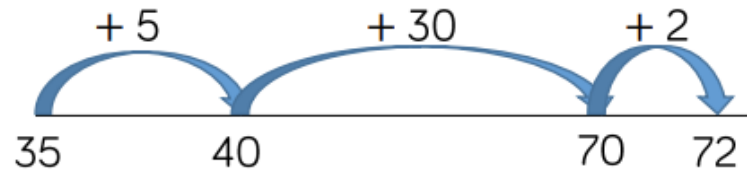
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

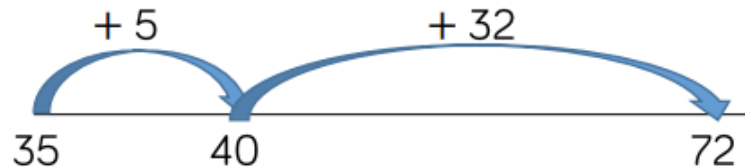
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

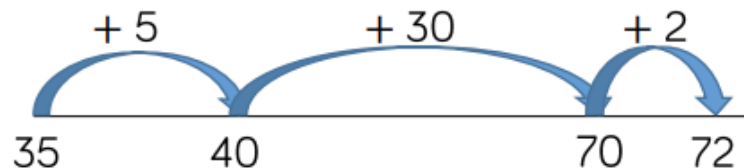
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

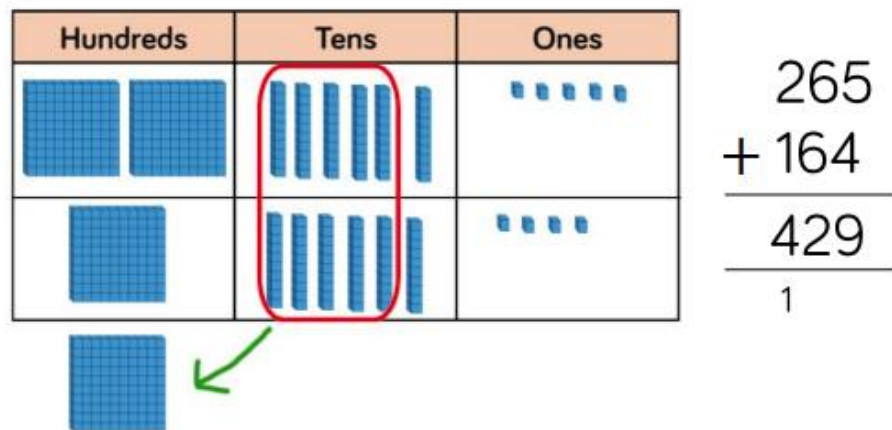
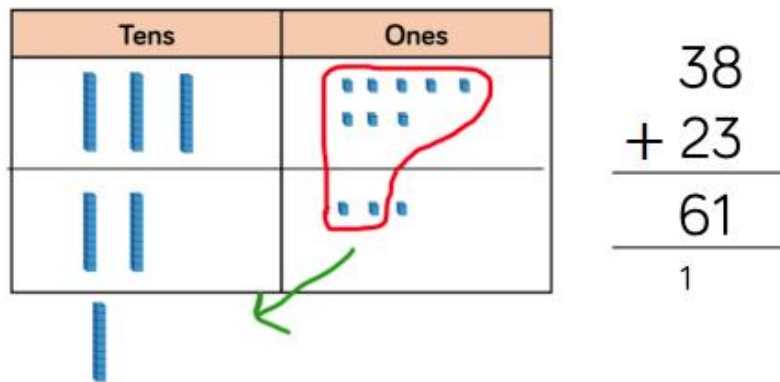
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (addition)



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

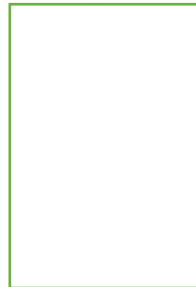
- How many ones are there altogether?
- Can we make an exchange? (Yes or No)
- How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
- How many ones do we have left? (Write in ones column)
- Repeat for each column.

Base 10/Dienes (subtraction)

Tens	Ones

$$\begin{array}{r} 5 1 \\ \cancel{6} 5 \\ - 28 \\ \hline 37 \end{array}$$

Hundreds	Tens	Ones



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)

Hundreds	Tens	Ones
100 100 100	10 10 10 10 10 10 10 10	1 1 1 1
100 100	10 10 10	1 1 1 1 1 1 1

100 10

$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 1 \quad 1 \end{array}$$

Ones	Tenths	Hundredths
1 1 1	0.1 0.1 0.1 0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01
1 1	0.1 0.1 0.1 0.1	0.01

1

$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)

Hundreds	Tens	Ones
		 

$$\begin{array}{r} 4 \quad 1 \\ 652 \\ - 207 \\ \hline 445 \end{array}$$

Thousands	Hundreds	Tens	Ones
	 		

$$\begin{array}{r} 3 \quad 1 \\ 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Year 3 Addition

Overview for addition:

Pupils should be taught to:

- add numbers mentally, including:
- a three-digit number and ones ($176+8$)
- a three-digit number and tens ($259 + 40$)
- a three-digit number and hundreds ($281 + 400$)
- add numbers with up to three digits, using formal written methods of columnar addition
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Number and place value skills needed for addition:

- read and write numbers up to 1000 in numerals and words
- recognise the place value of each digit in a three digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- count from 0 in multiples of 4, 8, 50 and 100: find 10 or 100 more or less than a given number.

solve number problems and practical problems involving these ideas.

Key Vocabulary: add, addition, more, plus, make, altogether, total, equal, double, most, count on, number line, hundreds, tens, ones, digit, partition, recombine, column, tens boundary, hundreds boundary, increase.

Adding numbers with up to three digits:

Progression:

TO + TO

HTO + TO - within tens boundary ($134 + 25$)

HTO + TO - crossing tens boundary ($235 + 48$)

HTO+ TO - crossing hundreds boundary ($483 + 35$)

HTO + TO - crossing tens and hundreds boundary ($488 + 47$)

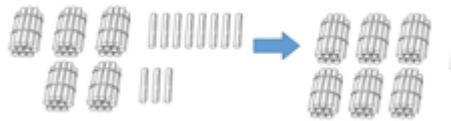
HTO + HTO - crossing tens boundary ($368 + 123$)

HTO + HTO - crossing tens and hundreds boundary ($576 + 477$)

Year 3 Addition

Add 2 digit numbers to 100

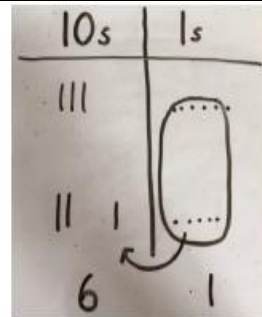
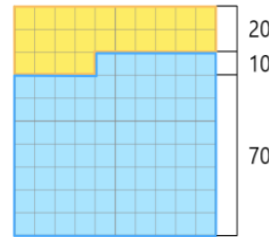
Concrete



Use of place value counters to add TO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

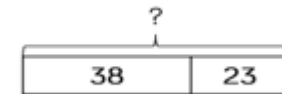
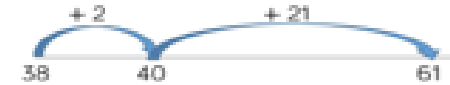
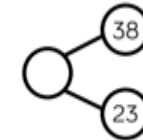
Pictorial

$$24 + 76 = 100$$



Children to represent the base 10 pictorially.

Abstract



$$38 + 23 = 61$$

Looking for ways to make 10.

$$\begin{array}{rcl}
 36 + 25 & = & 30 + 20 = 50 \\
 & & 5 + 5 = 10 \\
 & & 50 + 10 + 1 = 61
 \end{array}$$

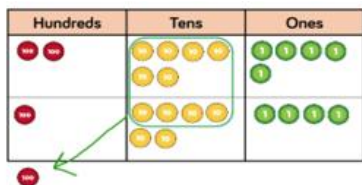
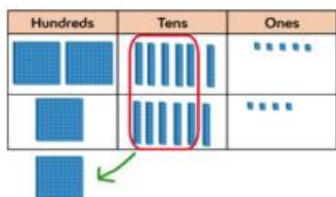
Formal method:

$$\begin{array}{r}
 +25 \\
 36 \\
 \hline
 61 \\
 1
 \end{array}$$

Year 3 Addition

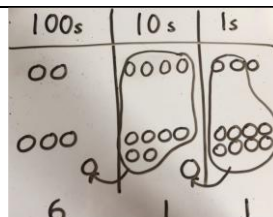
Add numbers with up to 3 digits

Concrete



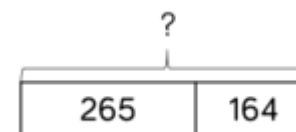
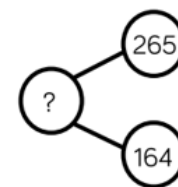
Use of place value counters/Dienes to add HTO + HTO etc.
When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

Pictorial



Children to represent the counters in a place value chart, circling when they make an exchange.

Abstract



$$265 + 164 = 429$$

$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$$

Year 4 Addition

Overview for addition:

Pupils should be taught to:

- add numbers with up to 4 digits using the formal written methods of column addition where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition two-step problems in contexts, deciding which operations and methods to use and why.
- solve simple measure and money problems with decimals to two decimal places.

Key vocabulary:

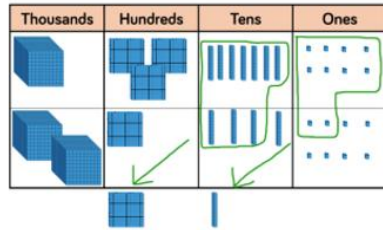
add, addition, more, plus, make, altogether, total, equal, double, most, count on, number line, hundreds, tens, ones, digit, partition, recombine, column, tens boundary, hundreds boundary, increase, horizontal expansion, carry, thousands, inverse, negative, decimal, tenths, hundredths

Number and place value skills needed for addition:

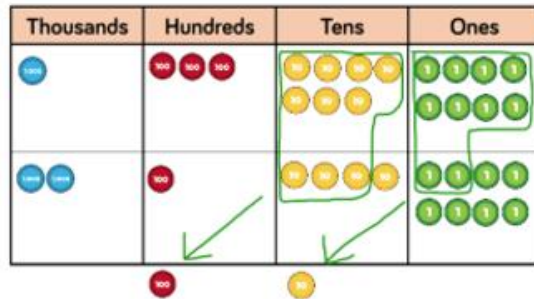
- recognise the place value of each digit in a four digit number (thousands, hundreds, tens and ones)
- order and compare numbers beyond 1000
- find 1000 more or less than a given number
- round any number to the nearest 10, 100, 1000
- count in multiples of 6, 7, 9, 25 and 1000
- count backwards through zero to include negative numbers
- solve number and practical problems that involve all of the above and with increasingly large positive numbers

Year 4 Add numbers with up to 4 digits

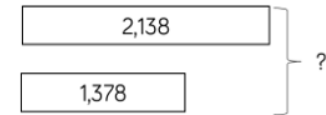
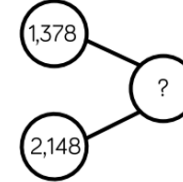
Concrete/Pictorial



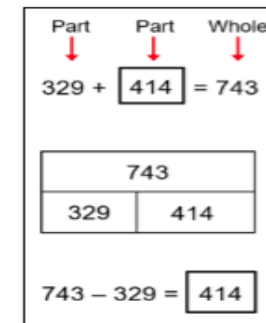
$$1378 + 2148$$



Abstract



$$\begin{array}{l} 300 + 60 + 7 \\ 100 + 80 + 5 \\ \hline 400 + 140 + 12 = 552 \end{array}$$



	1	3	7	8
+	2	1	4	8
	3	5	2	6
	1	1		

Year 5 Addition

Pupils should be taught to:

- add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- add numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why.
- solve problems involving numbers with up to three decimal places

Key vocabulary:

add, addition, more, plus, make, altogether, total, equal, double, most, count on, number line, hundreds, tens, ones, digit, partition, recombine, column, tens boundary, hundreds boundary, increase, horizontal expansion, carry, thousands inverse, negative, decimal, tenths, hundredths, thousandths

Number and place value skills needed for addition:

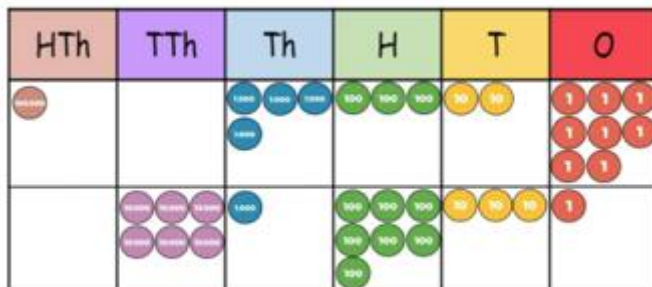
- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- solve number problems and practical problems that involve all of the above

Year 5 addition

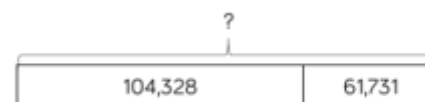
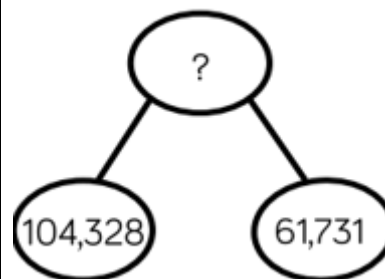
Progress to adding numbers with more than 4 digits, including money, measures and decimals with up to three decimal places

Concrete/Pictorial

$$104,328 + 61,731$$



Abstract



?	
104, 328	61, 731

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

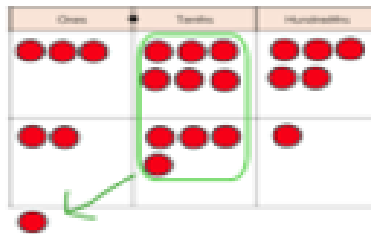
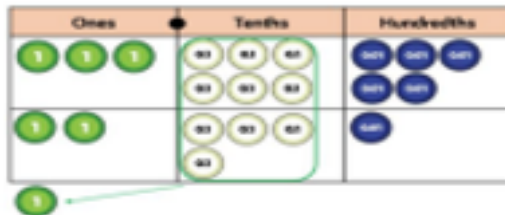
1

Year 5: Add numbers with up to 3 decimal places

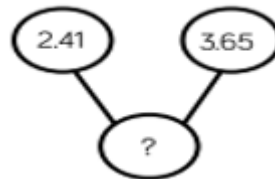
Progress to adding numbers with more than 4 digits, including money, measures and decimals with up to three decimal places

Concrete

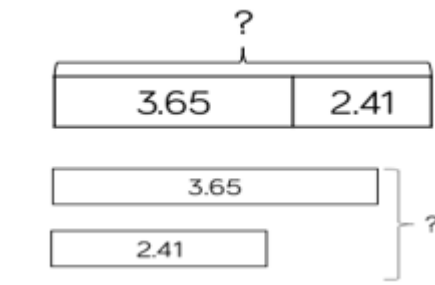
$$3.65 + 2.41$$



Abstract



?	
3.65	2.41



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

Year 6 Addition

Pupils should be taught to:

- perform mental calculations with mixed operations and large numbers.
- solve multi-step problems in contexts, deciding which operations and methods to use and why
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Key vocabulary:

add, addition, more, plus, make, altogether, total, equal, double, most, count on, number line, hundreds, tens, ones, digit, partition, recombine, column, tens boundary, hundreds boundary, increase, horizontal expansion, carry, thousands, inverse, negative, decimal, tenths, hundredths, thousandths

Number and place value skills needed for addition:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practical problems that involve all of the above.

Continue with models, images and methods from year 5 but with numbers up to 10 000 000

Year 6 will also use BODMAS which is an acronym to help children remember the order of mathematical operations.

B: Brackets

O: Order

D: Division

M: Multiplication

A: Addition

S: Subtraction

Year 3 Subtraction

Overview for subtraction:

Pupils should be taught to:

- subtract numbers mentally, including:
- a three-digit number and ones ($176 - 8$)
- a three-digit number and tens ($259 - 40$)
- a three-digit number and hundreds ($681 - 400$)
- subtract numbers with up to three digits.
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Key Vocabulary : take away, less, minus, subtract, subtraction, how many more/less, count back, difference, decrease, digit, hundreds, tens, ones,

Number and place value skills needed for subtraction:

- read and write numbers up to 1000 in numerals and words
- recognise the place value of each digit in a three digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- count from 0 in multiples of 4, 8, 50 and 100: find 10 or 100 more or less than a given number.
- solve number problems and practical problems involving these ideas.

Subtracting numbers with up to three digits:

Progression:

HTU - TO within tens boundary ($138 - 25$)

HTU - TO crossing tens boundary ($265 - 58$)

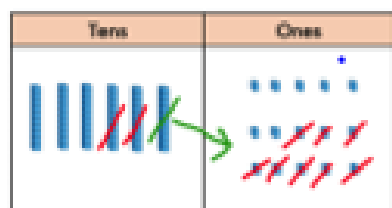
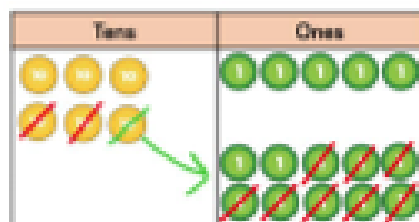
HTU – TO crossing hundreds and tens boundary ($265 - 78$)

HTU - HTO not crossing tens boundary ($365 - 123$)

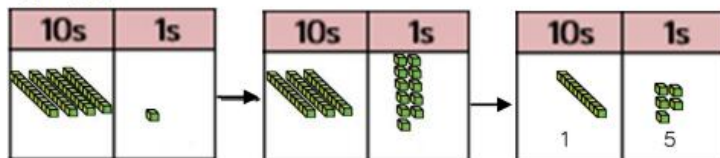
HTU - HTO crossing tens and hundreds boundary ($414 - 126$)

Year 3 Subtract 1 and 2 digit numbers to 100

Concrete

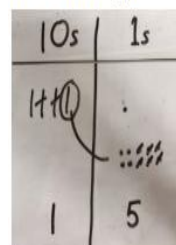


Column method using base 10 and having to exchange.
41 - 26

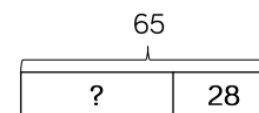
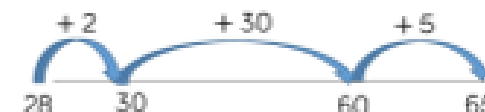
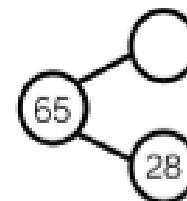


Pictorial

Represent the base 10 pictorially, remembering to show the exchange.



Abstract



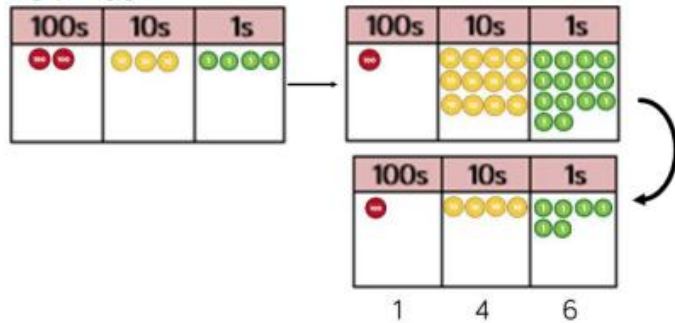
$$\begin{array}{r} 60 \\ 20 + 2 \\ - 40 + 7 \\ \hline 20 + 5 = 25 \end{array}$$

$$\begin{array}{r} 5 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$$

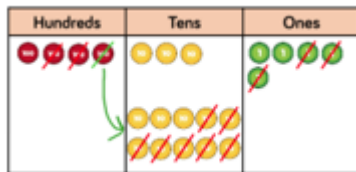
Year 3 Subtract numbers with up to 3 digits

Concrete

Column method using place value counters.
234 - 88

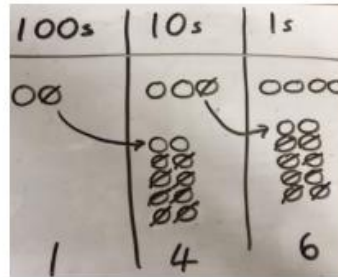


453 - 273



Pictorial

Represent the place value counters pictorially; remembering to show what has been exchanged



Abstract

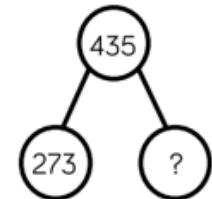
$$276 - 43 = 200 + 70 + 6 \\ - \quad 40 + 3 \\ \hline 200 + 30 + 3 = 233$$

Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$



$$\begin{array}{r} 435 \\ - 273 \\ \hline 162 \end{array}$$



Year 4 Subtraction

Pupils should be taught to:

- Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- solve simple measure and money problems with decimals to two decimal places.

Key vocabulary:

take away, less, minus, subtract, subtraction, how many more/less, count back, difference, decrease, digit, hundreds, tens, ones, partition, exchange, inverse, column, decomposition

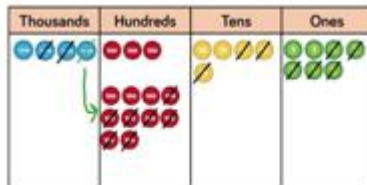
Number and place value skills needed for subtraction:

- recognise the place value of each digit in a four digit number (thousands, hundreds, tens and ones)
- order and compare numbers beyond 1000
- find 1000 more or less than a given number
- round any number to the nearest 10, 100, 1000
- count in multiples of 6,7,9,25 and 1000
- count backwards through zero to include negative numbers
- solve number and practical problems that involve all of the above and with increasingly large positive numbers

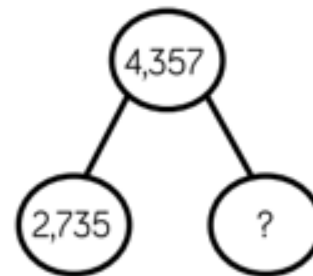
Year 4 Subtraction

Concrete/Pictorial

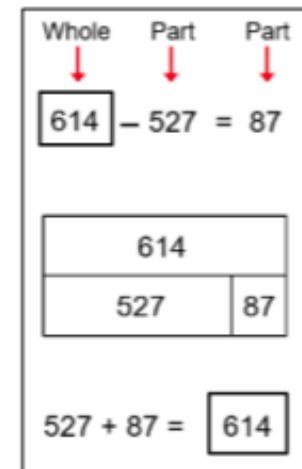
$$4357 - 2735$$



Abstract



$$\begin{array}{r}
 ^3 ^1 \\
 4357 \\
 - 2735 \\
 \hline
 1622
 \end{array}$$



Year 5 Subtraction

Pupils should be taught to:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- solve problems involving numbers with up to three decimal places

Key vocabulary:

take away, less, minus, subtract, subtraction, how many more/less, count back, difference, decrease, digit, hundreds, tens, ones, thousands, partition, exchange, inverse, column, decomposition, decimal, tenths, hundredths, thousandths

Number and place value skills needed for addition:

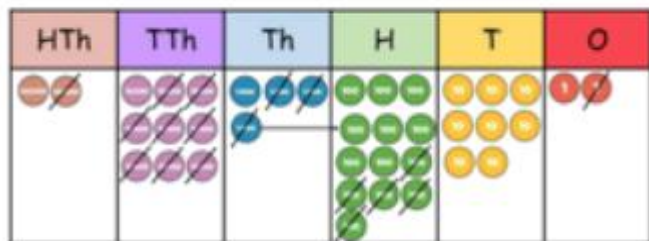
- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- solve number problems and practical problems that involve all of the above

Year 5 Subtraction

Subtract numbers with more than 4 digits

Concrete/Pictorial

$$294\,382 - 182\,501$$



Abstract



294 382	
182 501	?

	2	9	3	¹ 3	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Year 5 Subtraction

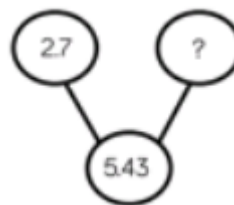
Subtract numbers with up to 3 decimal places

Concrete/Pictorial

5.43 - 2.7



Abstract



5.43	
2.7	?

$$\begin{array}{r} 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

Year 6 Subtraction

Pupils should be taught to:

- perform mental calculations with mixed operations and large numbers.
- solve multi-step problems in contexts, deciding which operations and methods to use and why
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Key vocabulary:

take away, less, minus, subtract, subtraction, how many more/less, count back, difference, decrease, digit, hundreds, tens, ones, thousands, partition, exchange, inverse, column, decomposition, compact, decimal, tenths, hundredths, thousandths

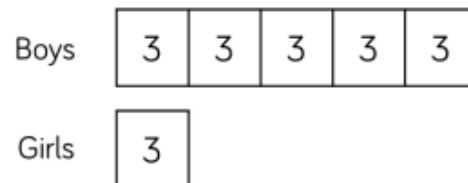
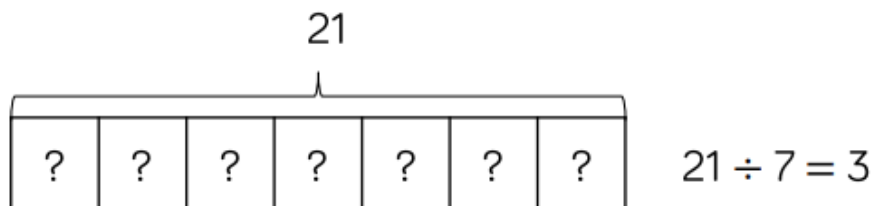
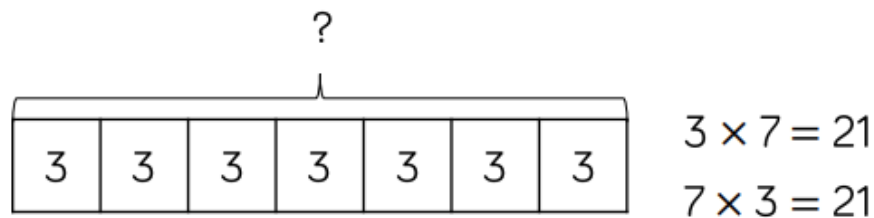
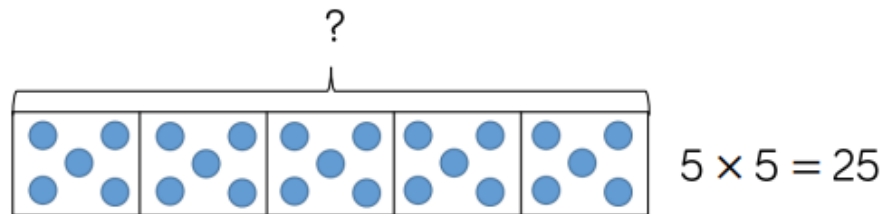
Number and place value skills needed for addition:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practical problems that involve all of the above.

Written methods:

Continue with models, images and methods from year 5 but progress to numbers up to 10 000 000 and mixed decimal numbers, aiming for both conceptual understanding and procedural fluency.

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.

Number Shapes



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

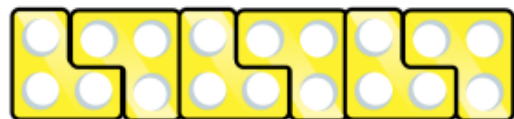


$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd $\times$ odd = even, odd $\times$ even = odd, even $\times$ even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Benefits

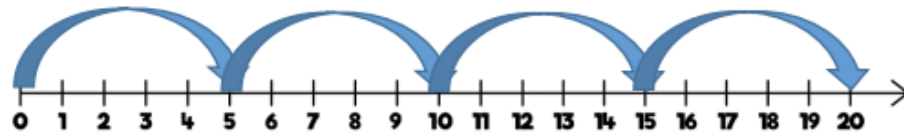
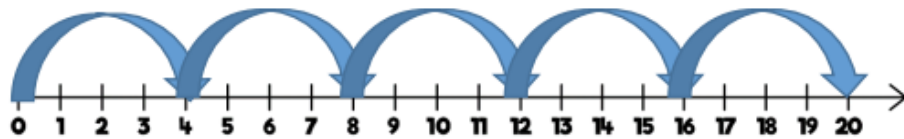
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

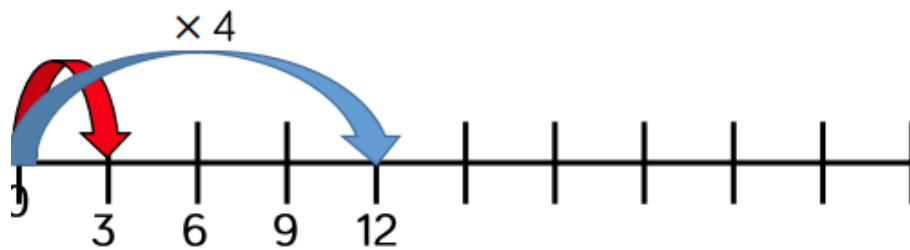
When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

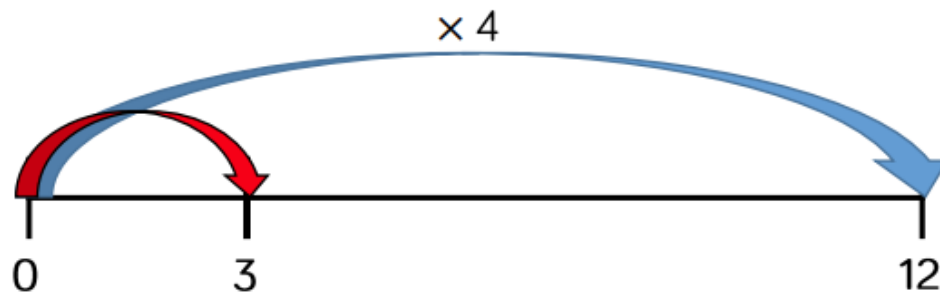
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

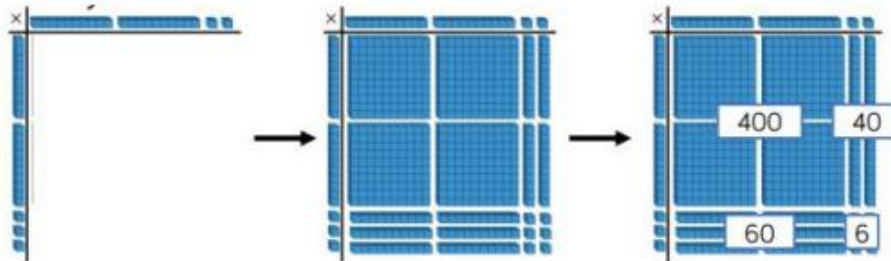
Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)

Hundreds	Tens	Ones

(A green box highlights the 12 ones in the 'Ones' column, and a green arrow points from it to the 'Tens' column, indicating an exchange of 10 ones for 1 ten.)

$$\begin{array}{r}
 24 \\
 \times 3 \\
 \hline
 72 \\
 \hline
 1
 \end{array}$$



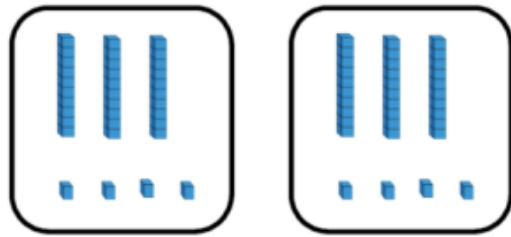
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

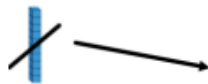
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Base 10/Dienes (division)

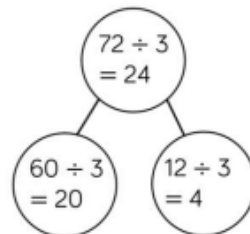


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)

Hundreds	Tens	Ones
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
100	10 10	

$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ 12 \end{array}$$

×	10 10 10 10	1 1 1 1
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
1	10 10 10 10	1 1 1 1
1	10 10 10 10	1 1 1 1

$$\begin{array}{r} 44 \\ \times 32 \\ \hline 8 \\ 80 \\ 120 \\ + 1200 \\ \hline 1408 \\ 1 \end{array}$$

Benefits

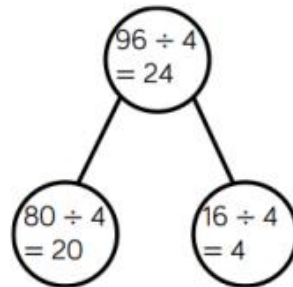
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)

Tens	Ones
10	1
10	1
10	1
10	1
10	1
10	1
10	1
10	1
10	1
10	1



Thousands	Hundreds	Tens	Ones
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000	100	10	1

$$4 \overline{) 1223} \\ \underline{4892} \\ 1$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

Year 3 Multiplication

Overview for multiplication:

Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.

Key Vocabulary : groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, column, row, commutative, times

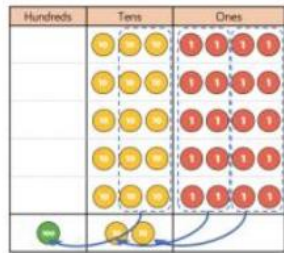
Key skills needed for multiplication:

- Partition numbers into hundreds, tens and ones
- Multiply multiples of 10 by a single digit (30×4)
- Recall multiplication facts for the 2,3,4,5,8 and 10 times tables.
- Know that multiplication is commutative (3×4 is the same as 4×3)
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)

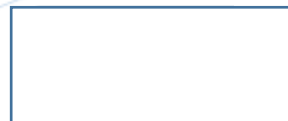
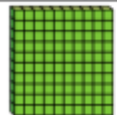
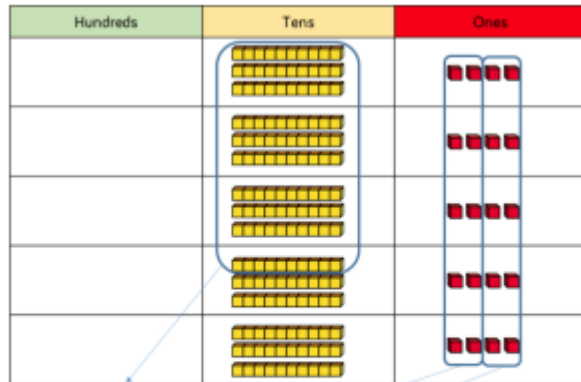
Year 3 Multiplication

Multiply 2 digit numbers by 1 digit numbers

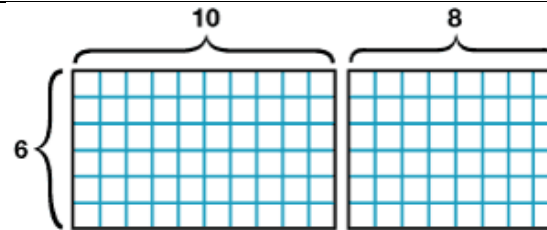
Concrete



$$34 \times 5$$

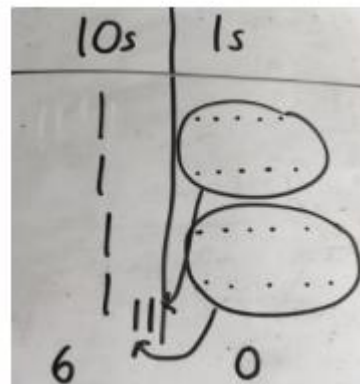


Pictorial

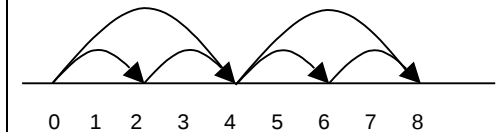


Use arrays to multiply 2 digit numbers by a single digit:

$$15 \times 4$$



Abstract



X	30	5	
2	60	10	=70

2x4 or 4x2

Children to record what it is they are doing to show understanding.

$$3 \times 23 = 3 \times 20 + 3 \times 3 = 60 + 9 = 69$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Year 4 Multiplication

Pupils should be taught to:

- recall multiplication facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1 multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit,

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, column, row, commutative, times, grid, multiple, product

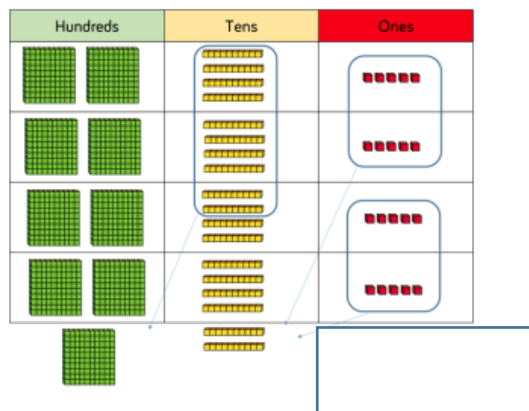
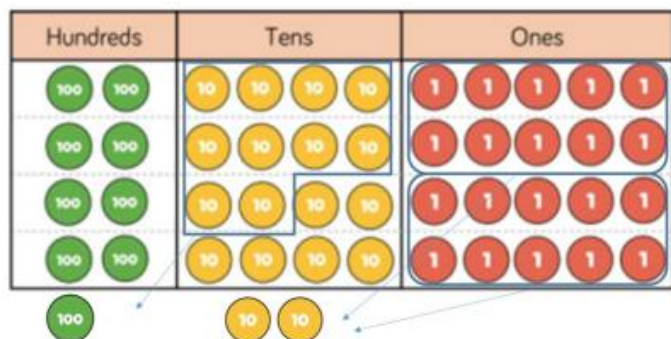
Key skills needed for multiplication:

- Recall all times-tables up to 12×12
- Recognise place value of digits in up to 4 digit numbers
- count in multiples of 6, 7, 9, 25 and 1000
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Year 4 Multiplication

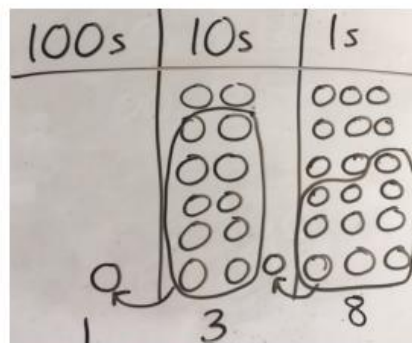
Multiply 2 digit and 3 digit numbers by a one digit number

Concrete



Pictorial

$$23 \times 6$$



Abstract

Recap grid method from year 3

X	30	5	
2	60	10	=70

Extend to formal written method.

	H	T	O	
x		3	4	
		2	0	(5 x 4)
+	1	5	0	(5 x 30)
	1	7	0	

	H	T	O
		3	4
x			5
	1	7	0
	1	2	

	H	T	O
	2	4	5
x			4
	9	8	0
	1	2	

Year 5 Multiplication

Pupils should be taught to:

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply numbers mentally drawing upon known facts
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- Solve problems involving multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, column, row, commutative, times, grid, multiple, product, inverse, square, cubed, factor, integer, decimal, short/long multiplication, carry, prime number,

Key skills needed for multiplication:

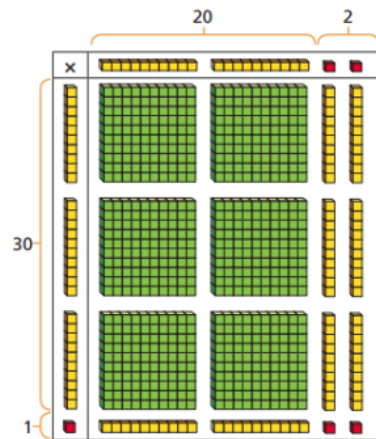
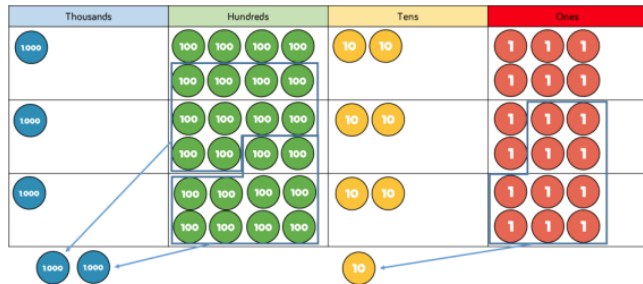
- know all multiplication tables up to 12 x 12
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit

Year 5 Multiplication

Multiply numbers up to 4 digits by a one- or two-digit number

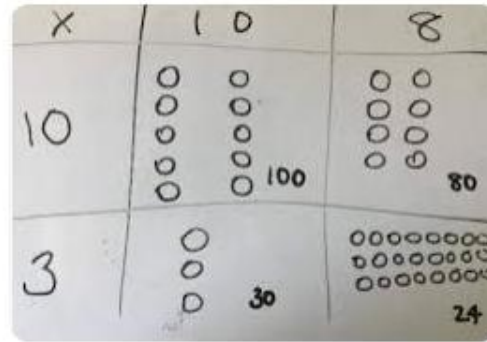
Concrete

$$1826 \times 3$$



Pictorial

$$18 \times 13$$



Abstract

$$1,826 \times 3 = 5,478$$

	Th	H	T	O
	1	8	2	6
x				3
	5	4	7	8

2 1

	H	T	O
		2	2
x		3	1
		2	2
	6	6	0
	6	8	2

$$\begin{array}{r} 2 \\ 24 \\ \times 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

Th	H	T	O
	2	3	4
x		3	2
	4	6	8
17	10	2	0
7	4	8	8

Year 6 Multiplication

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- identify common factors, common multiples and prime numbers
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine in the context of a problem, levels of accuracy.

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, column, row, commutative, times, grid, multiple, product, inverse, square, cubed, factor, integer, decimal, short/long multiplication, carry, prime number,

Key skills needed for multiplication:

- recall multiplication facts for all times-tables up to 12 x12 (as year 5 and year 5)
- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- perform mental calculations with mixed operations and large numbers.
- Round any integer to a required degree of accuracy.

Year 6 Multiplication

Multiply 4 digit numbers by 2 digit numbers

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2	1	9	1	2
2	5	3	7	
5	4	7	8	0
1		1		
7	6	6	9	2

1

When multiplying 4-digits by 2-digits, children should be confident in the written method.

Year 3 Division

Overview for division:

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers by one digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems involving division.

Key Vocabulary : share, share equally, group, equal groups, divide, arrays, divided by, divided into

Number and place value skills needed for division:

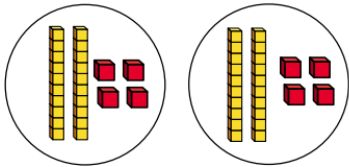
- Partition numbers into hundreds, tens and ones
- Recall multiplication facts for the 2,3,4,5,8 and 10 times tables.
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)

Year 3 Division

Divide 2 digits by 1 digit (progressing to sharing with exchange)

Concrete

$$48 \div 2 = 24$$



Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

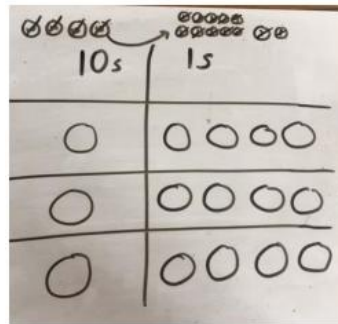
Sharing with exchange:

Tens	Ones
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1

$$52 \div 4 = 13$$

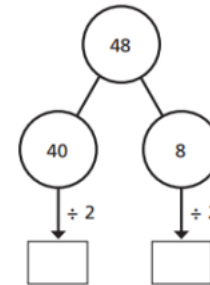
Pictorial

Children to represent the place value counters pictorially.



$$42 \div 3 = 14$$

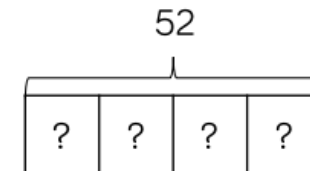
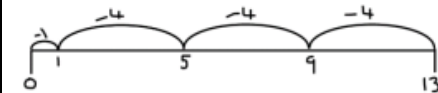
Abstract



$$13 \div 4 = 3 \text{ remainder } 1$$

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'



Year 4 Division

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to divide mentally, including: dividing by 1, 10 and 100
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

Key vocabulary: share, share equally, group, equal groups, divide, arrays, divided by, divided into, grouping, remainder, divisible by, factor

Number and place value skills needed for division:

- Recall all multiplication facts up to 12×12

Year 4 Division

Divide 2 and 3 digit numbers by a one digit number

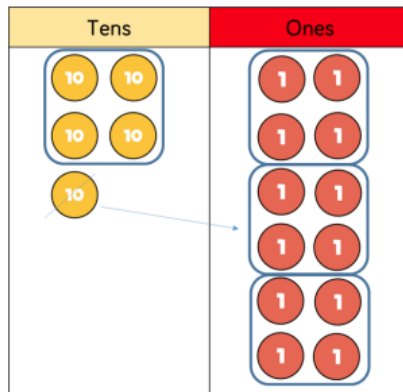
Concrete/Pictorial

$$53 \div 4 = 13 \text{ r}1$$

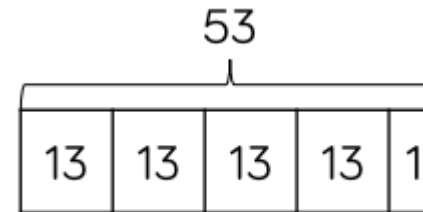
Sharing with remainders:



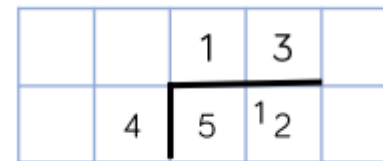
Grouping:



Abstract



When using the short division method children use grouping. Starting with the largest place value they group by the divisor.

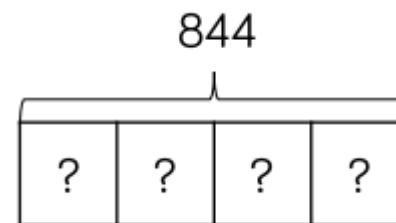
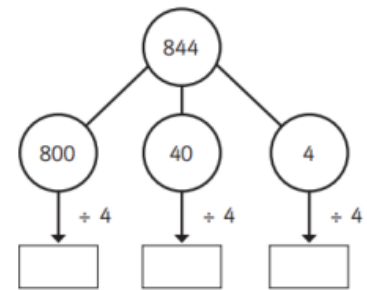


Sharing:

$$844 \div 4 = 122$$

H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1

Sharing:



Year 5 Division

Pupils should be taught to:

- ☐ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- ☐ divide whole numbers and those involving decimals by 10, 100 and 1000
- Solve problems involving division and a combination of these, including understanding the meaning of the equals sign
- ☐ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- ☐ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Key vocabulary: share, share equally, group, equal groups, divide, arrays, divided by, divided into, short division, multiple, inverse, quotient,

Number and place value skills needed for division:

- Know all multiplication facts up to 12×12

Year 5 Division

Divide numbers up to 4 digit numbers by a one digit number

Concrete

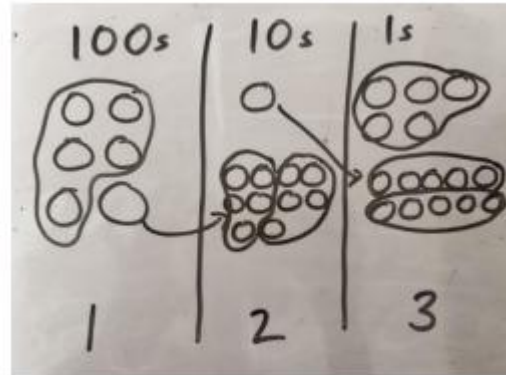
$$8,532 \div 2 = 4,266$$

Grouping:

Hundreds	Tens	Ones
100 100 100 100	10 10 10 10	1 1 1 1
100 100 100 100	10	1 1 1 1
		1 1 1 1
		1 1

Th	H	T	O
1,000 1,000	100 100	10 10	1 1
1,000 1,000	100 100	10	1 1
1,000 1,000	100	10 10	1 1
1,000 1,000		10 10	1 1
		10 10	1 1
		10 10	

Pictorial



Abstract

8532	
?	?

		2	1	4
	4	8	5	16

	4	2	6	6
2	8	5	13	12

Year 6 Division

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Key vocabulary:

share, share equally, group, equal groups, divide, arrays, divided by, divided into, short division, multiple, inverse, quotient,

Number and place value skills needed for addition:

- Know all multiplication facts up to 12×12

Year 6 Division

Divide numbers up to 4 digits by a two-digit whole number

Short division:

		0	3	6
	12	4	⁴ 3	⁷ 2

	0	4	8	9
15	7	⁷ 3	¹³ 3	¹³ 5

Children can write out multiples to support their calculations with larger remainders.

15	30	45	60	75	90	105	120	135	150
----	----	----	----	----	----	-----	-----	-----	-----

Extend to long division when ready.

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

$12 \times 1 = 12$
 $12 \times 2 = 24$
 $12 \times 3 = 36$
 $12 \times 4 = 48$
 $12 \times 5 = 60$
 $12 \times 6 = 72$
 $12 \times 7 = 84$
 $12 \times 8 = 96$
 $12 \times 9 = 108$
 $12 \times 10 = 120$

$$432 \div 12 = 36$$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	-	3	0	0	
			7	2	
	-		6	0	
			1	2	

Glossary

Addend – A number to be added to another.

Aggregation – combining two or more quantities or measures to find a total.

Array – an ordered collection of counters, cubes or other items in rows and columns.

Augmentation – increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Composite numbers – are whole numbers that have more than two factors.

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Distributive – if you multiply a number by two numbers that are being added together, it is the same as multiplying the number by each of the other numbers: $12 \times (3 + 5) = (12 \times 3) + (12 \times 5)$

Dividend – a dividend is the quantity that has to be divided.

Divisor – a divisor is a number that is divided into another.

Exchange – Change a number or expression for another of an equal value.

Equality - a symbolic expression of the fact that two quantities are equal.

Equation – an equation has two parts separated by an equals sign. The left part of the equation is always worth the same as the right part.

Fact Family - related addition and subtraction facts or multiplication and division facts, made from the same numbers.

Factor – a factor is a whole number that will divide exactly into another number.

Integer – an integer is any whole number. It can be a positive or negative number.

Inverse - the word inverse refers to the opposite of another operation.

Minuend – a quantity or number from which another is subtracted.

Multiple – every number has multiples. For example every number that 3 goes into is a multiple of 3.

Multiplicand – in multiplication, a number to be multiplied by another.

Operation – an operation is when you change a number by adding, subtracting, multiplying or dividing.

Partitioning – splitting a number into its component parts.

Product – the answer you get by multiplying two numbers together.

Quotient – a quotient is the answer to a division. It can be a whole number, fraction, mixed number or decimal.

Reduction – subtraction as take away.

Subitise – instantly recognise the number of objects in a small group without needing to count.

Subtrahend - a quantity or number to be subtracted from another.

Sum – the result of an addition.

Systematically - according to a fixed plan or system; methodically.

Total – the aggregate or the sum found by addition.